Math 131 Final Exam

Spring, 2022 – May 5, 2022

SOLUTIONS

Name: ____

Instructor Name: ____

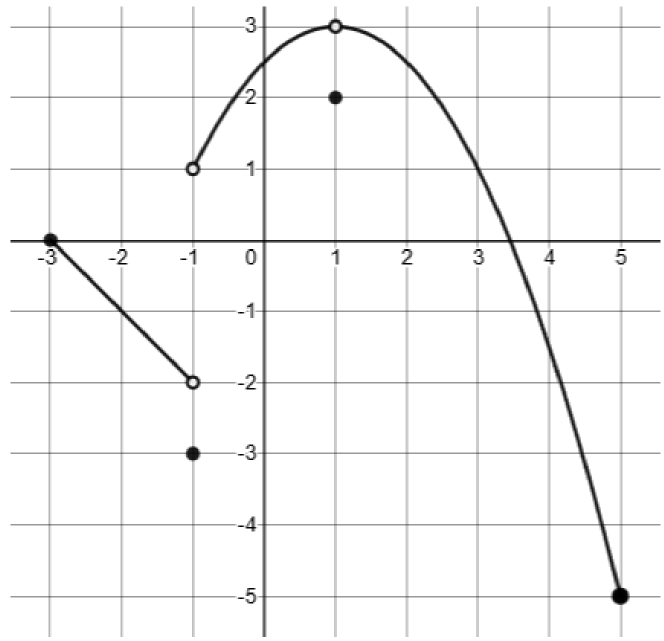
Time of class: ____

Page:	2	3	4	5	6	7	8	9	10	Total
Points:	16	9	16	8	10	12	6	16	7	100
Score:										

Instructions:

1. Do not open this exam until told to do so.
2. No books or notes may be used on the exam. There is an equation sheet on the last page of this exam.
3. For each problem, credit or partial credit will be given only when the appropriate explanation, work, steps, and/or algebra is shown so the grader can understand how you obtained the answer. Include units in your answer when possible.
4. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
5. Make sure your answer is clearly marked.
6. Read and follow directions carefully.
7. This exam has 15 questions, for a total of 100 points.There are 11 pages. Make sure you have them all.
8. You will have 120 minutes to complete the exam.
9. All cell phones and electronic devices (other than calculators) must be turned off during the exam.
10. Do not separate any of the pages of this exam except the last one containing the equation sheet. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
11. Calculators without internet access are allowed.

1) Use the figure to find each of the following. Write $-\infty$ or ∞ if the limit is negative or positive infinity. Write DNE if the limit is undefined.



(a) (2 $pts.$) $\lim_{x \to -1^+} f(x)$ $= 1$

(b) (2 $pts.$) $\lim_{x \to -1^-} f(x)$ $= -2$

(c) (2 $pts.$) $\lim_{x \to -1} f(x)$ $=$ DNE

(d) (2 $pts.$) $f(-1)$ $= -3$

2) Find each of the following limits exactly. If the limit is positive infinity, write ∞. If the limit is negative infinity, write $-\infty$. If the answer does not exist, write "DNE".

(a) (4 points) $\lim_{t \to \infty} \frac{100}{1+3e^{-t}}$ $= \frac{100}{1+0} = 100$

Reason

$\lim_{t \to \infty} (3e^{-t}) = \lim_{t \to \infty} \left(\frac{3}{e^{t}}\right) = 0$

(b) (4 points) $\lim_{t \to \infty} \frac{t^3}{2e^t}$ $= 0$

Reason

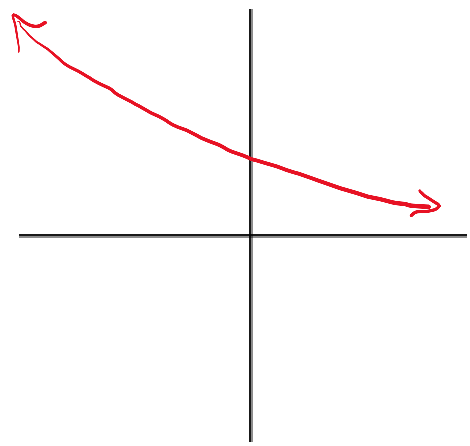
$2e^t$ Dominates t^3

3) (3 pts.) The function, $C = f(x)$, means the cost C, in cents, of selling x cups of coffee from a cart. What does $f\,'(20) = 25$ mean? (Select the best answer.)

a) It costs 25 cents to sell twenty cups of coffee.
b) The first twenty cups of coffee on average cost twenty-five cents each to sell.
c) After already selling twenty cups of coffee, the additional cost to sell the next cup of coffee is 25 cents.
d) The cost of selling coffee is increasing at a rate of 20 cents for every 25 cups of coffee sold.

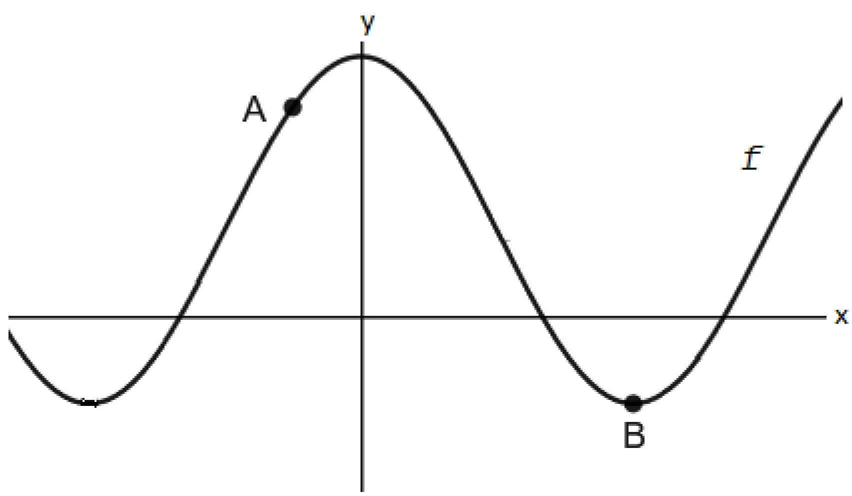
4) (3 pts.) Draw a graph, $y = f(x)$, that meets all the following conditions:

- the domain is all real numbers,
- $f(x)$ is always positive,
- the graph is everywhere decreasing,
- the graph is always concave up.



5) (0.5 pts. for each cell) On the table, fill in the signs (+ for positive, − for negative, 0 for zero) of f, f′, f″ at each marked point on the graph.

Point	f	$f\,'$	$f\,''$
A	+	+	−
B	−	0	+



6) (4 pts.) ATTENTION: This is the graph of $f\,'$, the derivative of f. It is NOT the graph of f. On which interval(s) is the function f increasing? Circle all that apply.

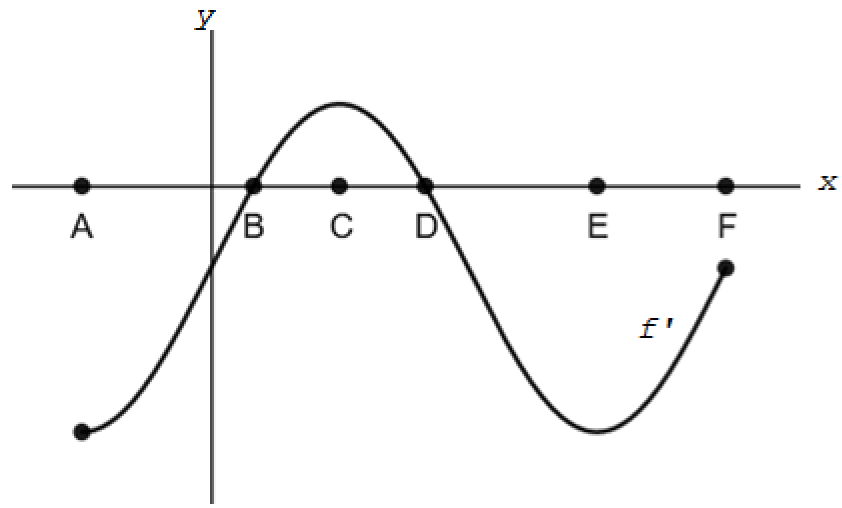
(i) AB

(ii) BC

(iii) CD

(iv) DE

(v) EF



7) Find the derivative of the following. Do not simplify.

(a) (3 points) $y = 4x^5 - 12x^3 + 6x + 1$

$$\boxed{\frac{dy}{dx} = 20x^4 - 36x^2 + 6}$$

(b) (3 points) $y = (3x - 6)^4 e^{5x}$

$f = (3x-6)^4 \qquad g = e^{5x}$

$f' \; 3 \cdot 4(3x-6)^3 \qquad g' = 5 \cdot e^{5x}$

$$\boxed{\frac{dy}{dx} = 12(3x-6)^3 \cdot e^{5x} + 5(3x-6)^4 \cdot e^{5x}}$$

(c) (3 points) $y = \cos^5(2x) + 7$

In $\cos(2x)$ $\qquad$ Out $(\cos(2x))^5$

$$\frac{dy}{dx} = -2\sin(2x) \cdot 5(\cos(2x))^4 = \boxed{-10\,(\sin(2x)) \cdot (\cos(2x))^4}$$

(d) (3 points) $y = \dfrac{\arctan(x)}{\ln(x)}$

$f = \arctan(x) \qquad g = \ln(x)$

$f' = \frac{1}{1+x^2} \qquad g' = \frac{1}{x}$

$$\boxed{\frac{dy}{dx} = \frac{\left(\frac{1}{1+x^2}\right)(\ln(x)) - (\arctan(x))\left(\frac{1}{x}\right)}{(\ln(x))^2}}$$

8) (8 points) Given the function

$$f(x) = x^3 - 6x^2 + 40$$

a) Find the first derivative and all the critical points.

$$f'(x) = 3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$x = 0$ and $x = 4$

b) On the interval $[-2, 5]$, find the x-values and y-values of the

(i) global maximum value

(ii) and global minimum value.

$f(-2) = 8$

$f(0) = 40$

$f(4) = 8$

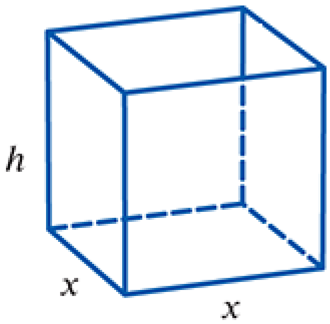
$f(5) = 15$

Global Maximum

$(0, 40)$

Global Minimums

$(-2, 8)$ and $(4, 8)$



9) (10 pts.) A manufacturer is inventing a new **open-top** box with a square base and wants to maximize the volume. The surface area of the box is fixed at 507 square inches. Use the diagram. Find *x* and *h* in inches. Give an exact answer.

x = 13

h = 6.5

Volume $= x \cdot x \cdot h = x^2 h$

Surface Area $= 507 = 4xh + x^2$

$507 - x^2 = 4xh$

$\frac{507 - x^2}{4x} = h$

$V = x^2\left(\frac{507 - x^2}{4x}\right) = \frac{507}{4}x - \frac{1}{4}x^3$

$\frac{dV}{dx} = \frac{507}{4} - \frac{3}{4}x^2 = 0$

$507 = 3x^2$

$x = \pm\sqrt{\frac{507}{3}} = 13$ $\quad h = \frac{507 - 13^2}{4(13)} = 6.5$

10) (6 pts.) You sell fancy cat toys for \$80 each. The total daily cost, $C(q)$, in dollars, of producing q units is given below. Find the maximum daily profit. Include units in your answer.

$$C(q) = 3q^2 + 8q + 7$$

$\pi(q) = R(q) - C(q)$

$= 80q - (3q^2 + 8q + 7)$

$= -3q^2 + 72q - 7$

$\pi'(q) = -6q + 72 = 0$

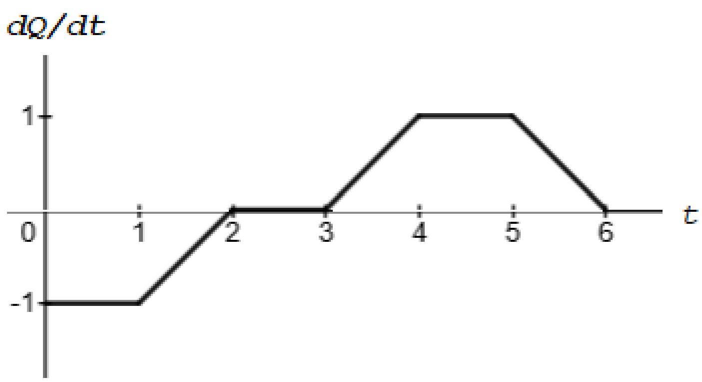
$-6q = -72$

$q = 12$ units

$\pi(12) = -3(12)^2 + 72(12) - 7$

$= \boxed{\$425}$

11) (1 pt. for each cell) Use the fact that $Q = 3$ when $t = 0$ and the figure below to fill in the table.

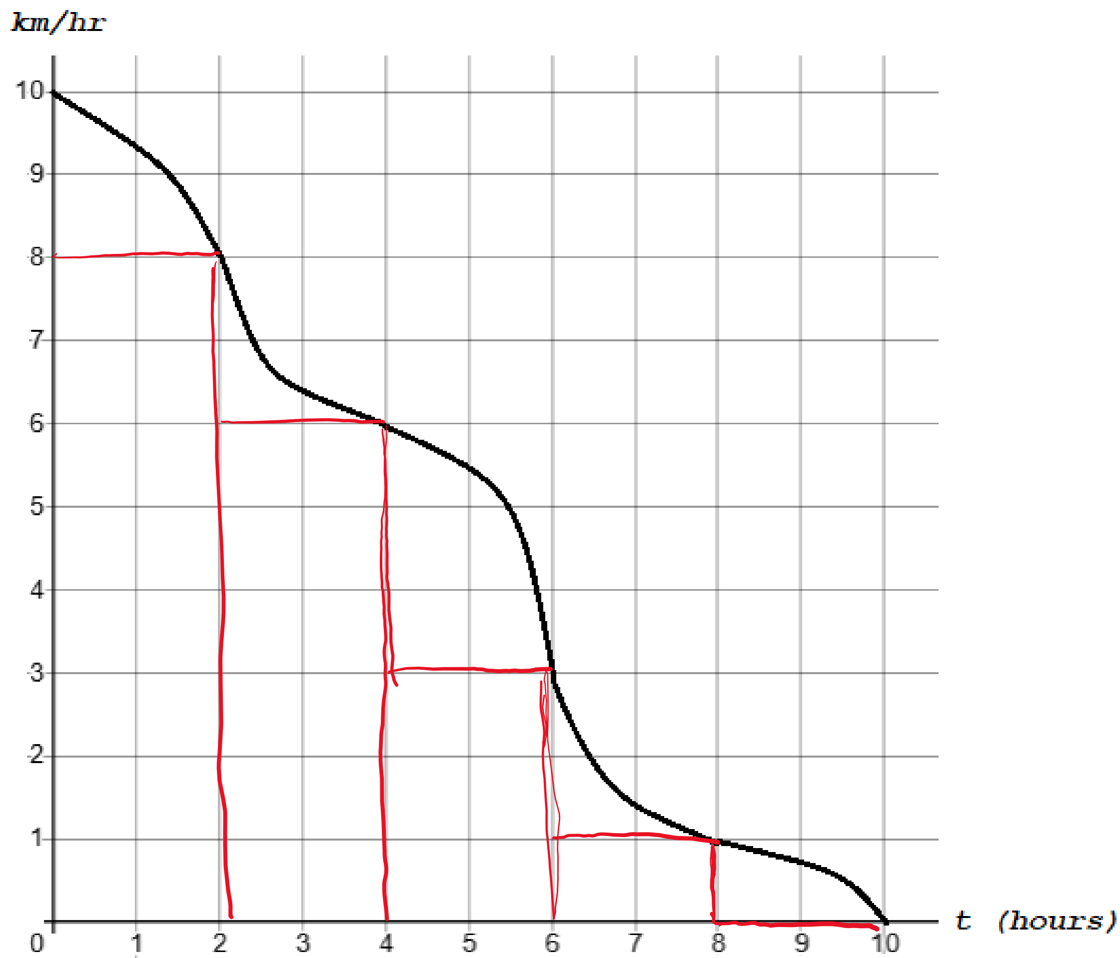


t	0	1	2	3	4	5	6
Q	3	2	1.5	1.5	2	3	3.5

12) The graph shows the velocity of an object for $0 \le t \le 10$.

a) (3 pts.) Draw rectangles representing a right-hand Riemann sum for the area under the curve with $\Delta t = 2$.

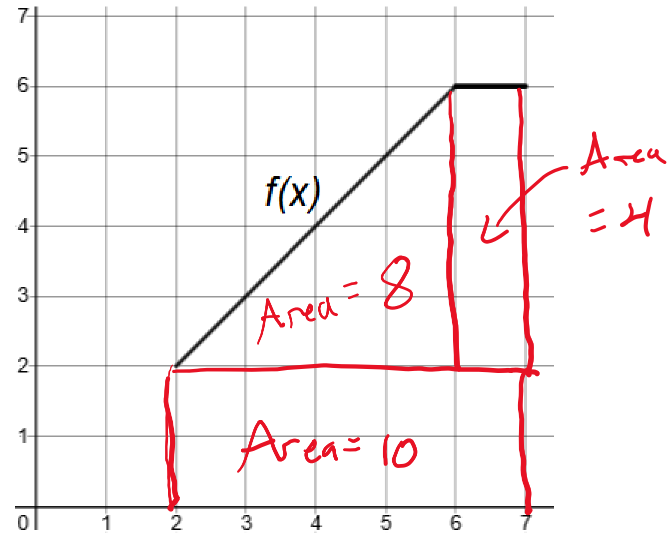
b) (3 pts.) Estimate the distance the object traveled by calculating the value of the sum. Include units in your answer.



Distance $= 2(8) + 2(6) + 2(3) + 2(1) + 2(0)$

$= \boxed{36 \text{ km}}$

13) The graph of $f(x)$ is given to the right.



(a) (4 points) Use the graph to compute $\int_2^7 f(x)\,dx$.

$= 10 + 8 + 4$

$= \boxed{22}$

(b) (4 points) What is the average value of f on $[2,7]$?

$= \dfrac{\int_2^7 f(x)\,dx}{7-2} = \dfrac{22}{5} = \boxed{4.4}$

14) Find the indefinite integral.

(a) (4 points) $\int \left(3\cos(x) - \frac{4}{x}\right) dx$ $= 3\sin(x) - 4 \cdot \ln(x) + C$

(b) (4 points) $\int e^{4x} + 5x^3\,dx$ $= \frac{1}{4}e^{4x} + \frac{5}{4}x^4 + C$

15) (7 points) At midnight there is already 5 cm of snow on the ground. Between between midnight and 4 a.m., it is snowing at the rate,

$$r(t) = 3\sqrt{t} + 4t\,,$$

in cm/hour, where t is the number of hours after midnight. How much total snow is on the ground at 4 a.m.? Include units in your answer.

$$\int_0^4 \left(3t^{\frac{1}{2}} + 4t\right) dt = \frac{3}{3/2} \cdot t^{3/2} + 2t^2 \Big|_0^4$$

$$= 2t^{3/2} + 2t^2 \Big|_0^4$$

$$= \left(2(4)^{3/2} + 2(4)^2\right) - \left(2(0)^{3/2} + 2(0)^2\right)$$

$$= 16 + 32 = 48 \text{ cm snow added}$$

$$48 + 5 = \boxed{53 \text{ cm snow on ground}}$$

Five derivative rules for operations on functions.

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Sum and Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}\left[f\big(g(x)\big)\right] = f'\big(g(x)\big) \cdot g'(x)$

Four Derivative Rules for Functions

Derivative of a Constant: $\frac{d}{dx}(c) = 0$.   The Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$.

Exponential Functions: General Case: $\frac{d}{dx}(a^x) = (ln\ (a)) \cdot a^x$

Exponential Functions: Special Case: $\frac{d}{dx}(e^x) = e^x$

Three Trigonometric Rules

$\frac{d}{dx}[\sin(x)] = \cos(x)$ $\frac{d}{dx}[\cos(x)] = -\sin(x)$ $\frac{d}{dx}[\tan(x)] = \sec^2(x) = \frac{1}{\cos^2(x)}$

Three Inverse Function Rules

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$ $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$ $\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

General Antiderivative Rules

If k is constant, $\int k\ dx = kx + C$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ when $n \neq -1$

$\int a^x dx = \frac{a^x}{\ln(a)} + C$

$\int e^x dx = e^x + C$

$\int \cos(x)\ dx = \sin(x) + C$

$\int \sin\ (x)\ dx = -\cos(x) + C$

$\int sec^2 x\ dx = \tan x + C$

$\int \frac{1}{x}\ dx = \ln(|x|) + C$

$\int \frac{1}{1+x^2}\ dx = \arctan(x) + C$

$\int \frac{1}{\sqrt{1-x^2}}\ dx = \arcsin(x) + C$